2.7 Inequalities

Variables: x, y, z

Real numbers: $\begin{cases} a, b, c, d \\ a_1, a_2, a_3, \ldots, a_n \end{cases}$, m, n

Determinants: D, D_x, D_y, D_z

126. Inequalities, Interval Notations and Graphs

Inequality	Interval Notation	Graph
$a \le x \le b$	$[a, b]$	
$a < x \le b$	$(a, b]$	
$a \le x < b$	$[a, b)$	
$a < x < b$	(a, b)	
$-\infty < x \le b$, $x \le b$	$(-\infty, b]$	
$-\infty < x < b$, $x < b$	$(-\infty, b)$	
$a \le x < \infty$, $x \ge a$	$[a, \infty)$	
$a < x < \infty$, $x > a$	(a, ∞)	

127. If $a > b$, then $b < a$.

128. If $a > b$, then $a - b > 0$ or $b - a < 0$.

129. If $a > b$, then $a + c > b + c$.

130. If $a > b$, then $a - c > b - c$.

131. If $a > b$ and $c > d$, then $a + c > b + d$.

132. If $a > b$ and $c > d$, then $a - d > b - c$.

133. If $a > b$ and $m > 0$, then $ma > mb$.

134. If $a > b$ and $m > 0$, then $\frac{a}{m} > \frac{b}{m}$.

135. If $a > b$ and $m < 0$, then $ma < mb$.

136. If $a > b$ and $m < 0$, then $\frac{a}{m} < \frac{b}{m}$.

137. If $0 < a < b$ and $n > 0$, then $a^n < b^n$.

138. If $0 < a < b$ and $n < 0$, then $a^n > b^n$.

139. If $0 < a < b$, then $\sqrt[n]{a} < \sqrt[n]{b}$.

140. $\sqrt{ab} \le \frac{a+b}{2}$,
where $a > 0$, $b > 0$; an equality is valid only if $a = b$.

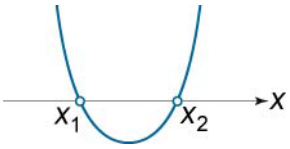
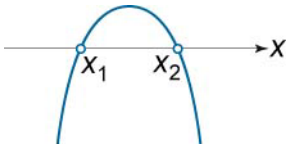
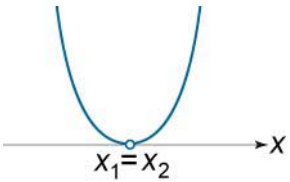
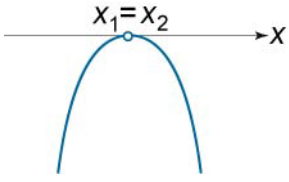
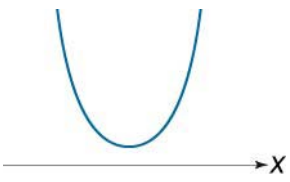
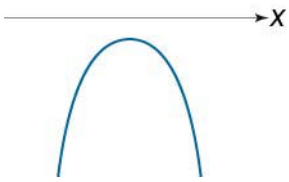
141. $a + \frac{1}{a} \ge 2$, where $a > 0$; an equality takes place only at $a = 1$.

142. $\sqrt[n]{a_1 a_2 \ldots a_n} \le \frac{a_1 + a_2 + \ldots + a_n}{n}$, where $a_1, a_2, \ldots, a_n > 0$.

143. If $ax + b > 0$ and $a > 0$, then $x > -\frac{b}{a}$.

144. If $ax + b > 0$ and $a < 0$, then $x < -\frac{b}{a}$.

145. $ax^2 + bx + c > 0$

	$a > 0$	$a < 0$
$D > 0$	 $x < x_1,\ x > x_2$	 $x_1 < x < x_2$
$D = 0$	 $x_1 < x,\ x > x_1$	 $x \in \varnothing$
$D < 0$	 $-\infty < x < \infty$	 $x \in \varnothing$

146. $|a+b| \le |a| + |b|$

147. If $|x| < a$, then $-a < x < a$, where $a > 0$.

148. If $|x| > a$, then $x < -a$ and $x > a$, where $a > 0$.

149. If $x^2 < a$, then $|x| < \sqrt{a}$, where $a > 0$.

150. If $x^2 > a$, then $|x| > \sqrt{a}$, where $a > 0$.

151. If $\frac{f(x)}{g(x)} > 0$, then $\begin{cases} f(x) \cdot g(x) > 0 \\ g(x) \neq 0 \end{cases}$.

152. $\frac{f(x)}{g(x)} < 0$, then $\begin{cases} f(x) \cdot g(x) < 0 \\ g(x) \neq 0 \end{cases}$.